3.3 product rule and quotient rule

2014-10-15 day 35

Product Rule

$$\frac{d}{dx}(fg) = f'g + fg'$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

3.3 product rule and quotient rule

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22) $y = \dfrac{3}{\sqrt{x}+2}$ ← I spy a bar.

I am a
a) an alcoholic
b) calc. student working on fractions
c) construction worker

$y' = \dfrac{f'g - fg'}{g^2}$ f = top, g = bottom

$= \dfrac{\frac{d}{dx}(3)\cdot(\sqrt{x}+2) - 3\left(\frac{d}{dx}(\sqrt{x}+2)\right)}{(\sqrt{x}+2)^2}$ $(x^{1/2}+2)$

could stop here $= \dfrac{(0)\cdot(\sqrt{x}+2) - 3\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{(\sqrt{x}+2)^2}$

$= \dfrac{-\frac{3}{2}\frac{1}{\sqrt{x}}}{(\sqrt{x}+2)^2} = \dfrac{-3}{2\sqrt{x}\,(\sqrt{x}+2)^2}$

$y'\big|_{x=1} = \dfrac{-3}{2\sqrt{1}(\sqrt{1}+2)^2}$

$= \dfrac{-3}{2(9)}$

$= -\dfrac{1}{6}$

3.3 product rule and quotient rule

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Many people have no options.
this inspires fear and despair.

The rest of us are taught [or learn]
that we have one option
That we have to find.

This is a lie.
We have many options. This should
inspire curiosity and reflection
- a sense of wonder.
Instead, this creates fear in us.
Learn to embrace uncertainty.

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25) $y = \frac{2x-1}{x+3}$

$$y' = \frac{f'g - fg'}{g^2}$$

$$y' = \frac{\frac{d}{dx}(2x-1)(x+3) - (2x-1)\frac{d}{dx}(x+3)}{(x+3)^2}$$

$$= \frac{2(x+3) - (2x-1)(1)}{(x+3)^2} = \frac{2x+6-2x+1}{(x+3)^2}$$

$$= \frac{7}{(x+3)^2} \text{ at } x=1, = \frac{7}{16}$$

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XXIX) $f(x) = x^3 - 3x + 1$

a) $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{(x^3 - 3x + 1) - (-1)}{x - 1}$

$= \dfrac{x^3 - 3x + 2}{x - 1} = \boxed{x^2 + x - 2}$

	x^3	x^2	x	x^0
(1)	1	0	-3	2
		1+	1+	-2+
	1	1	-2	0
	x^2	x	x^0	

b) $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$f'(1) = 3(1)^2 - 3 = 0$

3.3 product rule and quotient rule

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differentiating ≡

"take the derivative of"

differential eq^n ≡ eq^n involving derivatives

differential calculus ≡ calculus of derivatives

3.3 product rule and quotient rule

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3.4) $\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos(x)) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\left[\frac{\sin x}{\cos x}\right]$

$\frac{\frac{d}{dx}(\sin x)\cdot\cos x - \sin x\frac{d}{dx}(\cos x)}{(\cos x)^2}$

$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$ BTI $\frac{1}{\cos^2 x}$

$= \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$ $\left[\frac{1}{\cos x}\right]$

$= \frac{\frac{d}{dx}(1)\cos x - (1)\frac{d}{dx}(\cos x)}{\cos^2 x}$

$= \frac{0 - (-\sin x)}{\cos^2 x}$

$= \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}$

$= \sec x \tan x$

3.3 product rule and quotient rule

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$$\frac{d}{dx}(\cot x) = -\csc^2 x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\left[\frac{\cos x}{\sin x}\right] \qquad \left[\frac{1}{\sin x}\right]$$

$$= \frac{\frac{d}{dx}(\cos x)\cdot \sin x - \cos x \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$$

$$= -\csc^2 x$$

$$= \frac{\frac{d}{dx}(1)\cdot \sin x - (1)\frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$= \frac{-\cos x}{\sin^2 x}$$

$$= -\csc x \cot x$$

3.3 product rule and quotient rule

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3.4/5 $x^3 \sin x - 5\cos x$

x^3 TIMES sin x
product rule

constant times cos x
constant times derivative

$\frac{d}{dx}(x^3)\cdot \sin x + x^3 \cdot \frac{d}{dx}(\sin x) - 5\frac{d}{dx}\cos x$

$3x^2 \sin x + x^3(\cos x) - 5(-\sin x)$

$= 3x^2 \sin x + x^3 \cos x + 5\sin x$

$= (3x^2+5)\sin x + x^3 \cos x$

3.3 product rule and quotient rule

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6) $\dfrac{\cos x}{x\sin x}$ ← QUOTIENT

↑ PRODUCT

$$f'(x) = \frac{\frac{d}{dx}(\cos x)\cdot(x\sin x) - \cos x\cdot\frac{d}{dx}(x\sin x)}{(x\sin x)^2}$$

($\frac{d}{dx}(\cos x) = -\sin x$; $\frac{d}{dx}(x\sin x)$: PRODUCT RULE)

$$= \frac{-x\sin^2 x - \cos x\left[(1)\sin x + x\frac{d}{dx}(\sin x)\right]}{(x\sin x)^2}$$

($\frac{d}{dx}(\sin x) = \cos x$)

$$= \frac{-x\sin^2 x - \left[\sin x\cos x + x\cos^2 x\right]}{(x\sin x)^2}$$

$$= \frac{-\left(x\sin^2 x + x\cos^2 x + \sin x\cos x\right)}{(x\sin x)^2}$$

$$= \frac{-\left(x\left[\sin^2 x + \cos^2 x\right] + \sin x\cos x\right)}{(x\sin x)^2}$$

3.4/4) $f(x) = x^2 \cos x$

↑ product

$f'g + f g'$

$$f'(x) = \frac{d}{dx}(x^2) \cdot \cos x + x^2 \frac{d}{dx}(\cos x)$$

$[2x]$ $[-\sin x]$

$$= 2x\cos x - x^2 \sin x$$

7) $f(x) = \sec x - \sqrt{2}\tan x$

const times primitive

$f'(x) = \sec x \tan x - \sqrt{2}\sec^2 x$

$= \sec x(\tan x - \sqrt{2}\sec x)$

VIII $f(x) = (x^2+1)\sec x$

product

$f'(x) = \frac{d}{dx}(x^2+1)\cdot \sec x + (x^2+1)\cdot \frac{d}{dx}\sec x$

$= (2x)\sec x + (x^2+1)(\sec x \tan x)$

$= \sec x(2x + (x^2+1)\tan x)$

p.380

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

9) $f(x) = \sec x \tan x$

↑ product

#L #L
1D10T

$f'(x) = \frac{d}{dx}(\sec x)\tan x + \sec x \frac{d}{dx}(\tan(x))$

$= (\sec x \tan x)\tan x + \sec x(\sec^2 x)$

$= \sec x \tan^2 x + \sec^3 x$

$= \sec x(\tan^2 x + \sec^2 x)$

3.3/ 33-37, 39-40, 43-44, 45, 47,
49, 51, 57-60, 69, 71, 75-76, 80

3.4/ 25-27, 31-33, 36, 43

12) $f(x) = x - 4\csc x + 2\cot x$

const. times — const times —

$f'(x) = 1 + 4(\csc x \cot x) + 2(-\csc^2 x)$

$= 1 + 4\csc x \cot x - 2\csc^2 x$

ashley equals NOOB x+∞

☆ $\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\tan^2 x + 1 = \sec^2 x$